PHYS 301
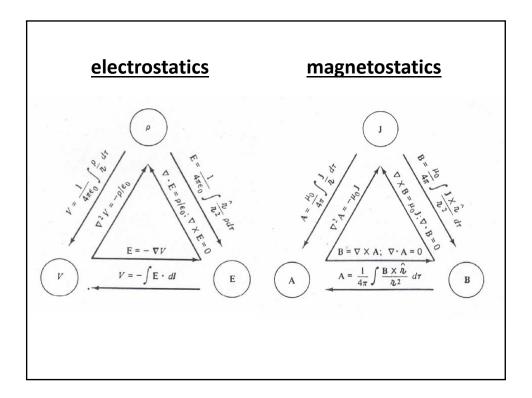# Electricity and Magnetism

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Today!

➢Magnetic fields
  ➢Force Law
  ➢Law of Biot-Savart
  ➢Ampere's Law

electrostatics   magnetostatics

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magnetostatics

THE LORENTZ FORCE LAW

· In the presence of both electric and magnetic fields, $\vec{E}$ and $\vec{B}$, a charge, Q will experience a force
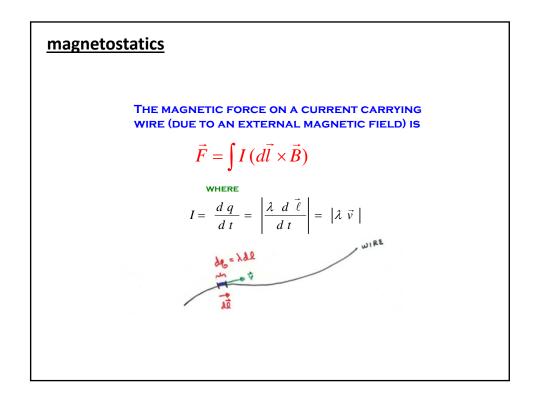
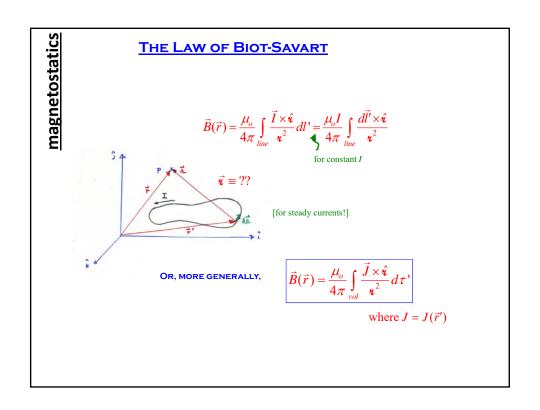$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

where $\vec{v}$ is the velocity of the charge, Q.

· NOTE THAT …

Magnetic fields do no work!

They may alter the direction a particle moves

but they cannot – but they cannot alter its speed!

magnetostatics

THE MAGNETIC FORCE ON A CURRENT CARRYING WIRE (DUE TO AN EXTERNAL MAGNETIC FIELD) IS

$$\vec{F} = \int I \left(\vec{dl} \times \vec{B} \right)$$

WHERE

$$I = \frac{dq}{dt} = \left| \frac{\lambda \, d \, \vec{\ell}}{dt} \right| = \left| \lambda \, \vec{v} \right|$$

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magnetostatics

THE LAW OF BIOT-SAVART

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{line} \frac{\vec{I} \times \hat{\imath}}{\imath^2} dl' = \frac{\mu_o I}{4\pi} \int_{line} \frac{\vec{dl'} \times \hat{\imath}}{\imath^2}$$

for constant I

$$\vec{\imath} \equiv ??$$

[for steady currents!]

OR, MORE GENERALLY,

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \int_{vol} \frac{\vec{J} \times \hat{\imath}}{\imath^2} d\tau'}$$

where $J = J(\vec{r}')$

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THE DIVERGENCE OF $\vec{B}$

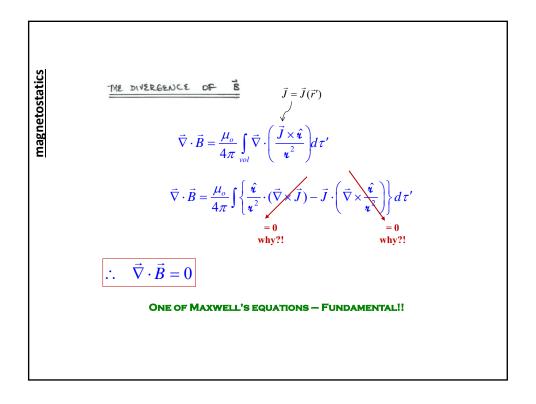**Expectations:**   "locally," the magnetic field for a current-carrying wire looks like this:

? WHAT'S THE DIRECTION OF $\vec{J}$, HERE? $\longrightarrow$ ▷

? WHAT'S AN EXPRESSION FOR $\vec{J}(\vec{r})$?

[since $\vec{J} = d\vec{I}\,/\,dA_{\perp}$, then $I = \int\limits_{surface} \vec{J}\cdot\hat{n}\,dA$]

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magnetostatics

THE DIVERGENCE OF $\vec{B}$

$\vec{J} = \vec{J}(\vec{r}')$

$$\vec{\nabla}\cdot\vec{B} = \frac{\mu_o}{4\pi}\int\limits_{vol}\vec{\nabla}\cdot\left(\frac{\vec{J}\times\hat{\imath}}{\imath^2}\right)d\tau'$$

$$\vec{\nabla}\cdot\vec{B} = \frac{\mu_o}{4\pi}\int\left\{\frac{\hat{\imath}}{\imath^2}\cdot(\vec{\nabla}\times\vec{J}) - \vec{J}\cdot\left(\vec{\nabla}\times\frac{\hat{\imath}}{\imath^2}\right)\right\}d\tau'$$

= 0
why?!   = 0
why?!

$$\therefore\quad \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

ONE OF MAXWELL'S EQUATIONS — FUNDAMENTAL!!

magnetostatics

$$\therefore \vec{\nabla} \times \vec{B} = \frac{\mu_0}{4\pi} \cdot 4\pi \vec{J}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

AMPERE'S LAW — ONE OF MAXWELL'S EQUATIONS.

* IT'S ALWAYS TRUE — BUT NOT ALWAYS USEFUL!